An Interacting Dark Energy Model for the Expansion History of the Universe

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We explore a model of interacting dark energy where the dark energy density is related by the holographic principle to the Hubble parameter, and the decay of the dark energy into matter occurs at a rate comparable to the current value of the Hubble parameter. We find this gives a good fit to the observational data supporting an accelerating Universe, and the model represents a possible alternative interpretation of the expansion history of the Universe.

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I. INTRODUCTION

Observations of type Ia supernova indicate that the Universe is making a transition from a decelerating phase to an accelerating one. The conventional explanation for this behavior is that a dark energy component is coming to dominate over and the matter-dominated phase is giving way to a phase dominated by a dark energy component. In fact a good fit is obtained from the observational data by assuming the cosmological model (ΛCDM) involving a cosmological constant Λ and cold dark matter in about the ratios $\Omega_{m,0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$.

In this paper we show that one can describe the cosmological data with a model of dark energy that includes an interaction that effects a transition (decay) of dark energy into matter. The model incorporates a holographic principle[1, 2] to determine the dark energy density of the Universe. The principle relates the dark energy scale to the Hubble horizon which has been the subject of speculation for applying holographic ideas to cosmology[3, 4, 5, 6, 7, 8, 9, 10]. This is a common choice for imposing holography on cosmology, and it has the most natural thermodynamic interpretation. However, this conditon imposed on the dark energy yields an equation of state that is matter-like[11] and therefore inconsistent with observational data. Subsequently efforts have been made to impose a holographic constraint based on some other physical horizon such as the particle horizon[12, 13] or the future event horizon[14, 15, 16, 17, 18, 19, 20, 21] or some other physical condition[22, 23, 24, 25, 26, 27, 28].

A suitable evolution of the Universe is obtained when, in addition to the holographic dark energy, an interaction (decay of dark energy to matter) is assumed, and the decay rate should be set roughly equal to the present value of the Hubble parameter for a good fit to the expansion history of the Universe as determined by the supernova and cosmic microwave background (CMB) data.

Interacting dark energy has been studied previously[29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]

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primarily with a goal of explaining the cosmic coincidence problem. A survey of the possible dynamics of dark energy can be found in Ref. [42]. In the presence of an interaction the dark energy can achieve a stable equilibrium that differs from the usual de Sitter case or the approach to the stable equilibrium can be made more gradual. Such models offer the hope of solving the coincidence problem that exists in the ΛCDM model where there is no obvious reason why the transition from matter domination to domination by the dark energy is occuring during the current epoch.

This paper presents a simple model that gives an acceptable expansion history of the Universe in terms of a holographic condition on the dark energy that relates its size to the Hubble scale. This is a common choice for imposing holography on cosmology, and it has the most natural thermodynamic interpretation. The effective equations of state of matter and dark energy coincide and behave like cold dark matter (CDM) at early times. The transition to behavior like a cosmological constant is effected by simply assuming there is a constant decay of dark energy into matter. The coincidence problem appears in the interacting dark energy model as the choice of the scale for the dark energy interaction which must be close to the present value of the Hubble parameter.

II. A FRAMEWORK FOR INTERACTING DARK ENERGY

The continuity equations for dark energy and matter are

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q ,$$
$$\dot{\rho}_m + 3H(1 + w_m)\rho_m = Q . \qquad (1)$$

The interaction is given by the quantity Q, and the energy-momentum tensor remains conserved as long as the same factor Q appears on the right hand side of each equation. To appeal to our intuition and to facilitate comparison with the more familiar case of ΛCDM model, it is useful to define effective equations of state that play the role of the native equations of state when an interac-

tion is present. Following Ref. [20], if we define

$$w_\Lambda^{\text{eff}} = w_\Lambda + \frac{\Gamma}{3H} , \qquad w_m^{\text{eff}} = w_m - \frac{1}{r}\frac{\Gamma}{3H} . \qquad (2)$$

where $\Gamma = Q/\rho_\Lambda$ is a rate, the continuity equations can be written in their standard form

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{\text{eff}})\rho_\Lambda = 0 ,$$
$$\dot{\rho}_m + 3H(1 + w_m^{\text{eff}})\rho_m = 0 \qquad (3)$$

Define as usual

$$\Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3M_p^2 H^2} , \qquad \Omega_m = \frac{8\pi\rho_m}{3M_p^2 H^2} . \qquad (4)$$

If we restrict our attention to the flat case,

$$\Omega_\Lambda + \Omega_m = 1 , \qquad (5)$$

one obtains a differential equation

$$\frac{d\Omega_\Lambda}{dx} = -3\Omega_\Lambda(1 - \Omega_\Lambda)\left[w_\Lambda^{\text{eff}} - w_m^{\text{eff}}\right] . \qquad (6)$$

where $x = \ln(a/a_0)$ and a is the Friedmann-Robertson-Walker (FRW) scale factor, and the "0" subscript here and on other quantities indicates the parameter value at the present time. For the ΛCDM model, where $w_\Lambda^{\text{eff}} = w_\Lambda = -1$ and $w_m^{\text{eff}} = w_m = 0$, the solution is the familiar one which can drive a small vacuum energy solution from a fixed point at $\Omega_\Lambda = 0$ toward a de Sitter Universe fixed point at $\Omega_\Lambda = 1$. In the presence of a holographic principle and/or an interaction, the effective equations of state can depend on x, and there is the possibility that another fixed point can develop at a value of Ω_Λ other than 0 or 1.

As shown in Ref. [40] two physical assumptions are required to determine the evolution of Ω_Λ and Ω_m. These can be chosen from the following list: (a) a "holographic principle" which specifies ρ_Λ in terms of some length scale associated with a horizon, (b) an assumption for the interaction Γ, or (c) an assumption about the native equations of state w_Λ and w_m. The assumptions commonly used for (a) are assumptions for the length scale L_Λ in

$$\rho_\Lambda = \frac{3c^2 M_p^2}{8\pi L_\Lambda^2} . \qquad (7)$$

Here the constant c represents an order one constant. Some choices have been to identify the length scale with a physical length such as the Hubble horizon, the particle horizon, or the future event horizon, or perhaps some other parameter. The interaction rate in (b) can be specified by an assumption of how the ratio of rates Γ/H varies as a function of Ω_Λ. The assumption for the equations of state in (c) have been largely confined to constant ones.

It is important to note that specifying two of the three conditions determines the third. For example a generic choice for the length scale L_Λ and for the interaction rate

Γ is inconsistent with a constant equation of state. It is easy to show that[40]

$$\Gamma = 3H(-1 - w_\Lambda) + 2\frac{\dot{L}_\Lambda}{L_\Lambda} . \qquad (8)$$

In addition to providing the connection between the various rates and the equation of state, this also establishes that the interaction should be of the size of H.

Using the above formulism one can show that (assuming $w_m = 0$)

$$w_\Lambda^{\text{eff}} = -1 + \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda} . \qquad (9)$$

$$w_m^{\text{eff}} = -\frac{\Gamma}{3H}\frac{\Omega_\Lambda}{(1 - \Omega_\Lambda)} . \qquad (10)$$

Then one obtains from Eq. (6) the differential equation

$$\frac{d\Omega_\Lambda}{dx} = 3\Omega_\Lambda(1 - \Omega_\Lambda)\left[1 - \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda} - \frac{\Gamma}{3H}\frac{\Omega_\Lambda}{(1 - \Omega_\Lambda)}\right] \quad (11)$$

This evolution equation is general and is obviously most useful when an assumption for the holographic principle and for the interaction are specified. For a constant equation of state together with a holographic principle the evolution equation becomes

$$\frac{d\Omega_\Lambda}{dx} = 3\Omega_\Lambda\left[1 + w_\Lambda\Omega_\Lambda - \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda}\right] \qquad (12)$$

On the other hand if a constant equation of state and the interaction is specified, one can eliminate the explicit dependence on $\dot{L}_\Lambda/L_\Lambda$ and obtain

$$\frac{d\Omega_\Lambda}{dx} = 3\Omega_\Lambda(1 - \Omega_\Lambda)\left[-w_\Lambda - \frac{\Gamma}{3H}\frac{1}{(1 - \Omega_\Lambda)}\right] . \qquad (13)$$

Finally the evolution of the Hubble parameter is given by

$$\frac{\dot{H}}{H^2} = \frac{1}{H}\frac{dH}{dx} = -\frac{3}{2}(1 - \Omega_\Lambda) - \frac{\Omega_\Lambda}{H}\frac{\dot{L}_\Lambda}{L_\Lambda} + \frac{\Gamma}{2H}\Omega_\Lambda . \quad (14)$$

Equations (11) and (14) then determine a set of equations that give $\Omega_\Lambda(z)$ and $H(z)$ where z is the redshift, and include the usual ΛCDM model as a special case.

III. INTERACTING DARK ENERGY MODEL

The ΛCDM model is defined in terms of the preceding formalism as

$$\frac{\dot{L}_\Lambda}{L_\Lambda} = 0 , \qquad \Gamma = 0 . \qquad (15)$$

The equations of state of matter and dark energy are 0 and -1 respectively, and the model predicts an evolution of the Hubble parameter as

$$H(z) = H_0\left[(1 + z)^3\Omega_{m,0} + \Omega_{\Lambda,0}\right]^{1/2} , \qquad (16)$$

where $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ are the present values of matter and dark energy. A good fit to the supernova data is given by $\Omega_{m,0} \approx 0.3$ and $\Omega_{\Lambda,0} \approx 0.7$.

Various physical assumptions have appeard for the holographic condition on $\dot{L}_\Lambda/L_\Lambda$. The cosmological constant (ΛCDM) requires $\dot{L}_\Lambda/L_\Lambda = 0$. Other conditions involve invoking a holographic arguement such as relating the length scale to various physical horizons. For example, one can relate L_Λ to the Hubble horizon (HH), the particle horizon (PH), or the future horizon (FH) for which

$$
\begin{aligned}
R_{\text{HH}} &= 1/H \,, \\
R_{\text{PH}} &= a \int_0^t \frac{dt}{a} = a \int_0^a \frac{da}{Ha^2} \,, \\
R_{\text{FH}} &= a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2} \,.
\end{aligned}
\tag{17}
$$

These yield

$$
\begin{aligned}
\frac{1}{H}\frac{\dot{L}_\Lambda}{L_\Lambda} &= -\frac{\dot{H}}{H^2} \quad \text{(HH)} \,, \\
&= 1 + \frac{\sqrt{\Omega_\Lambda}}{c} \quad \text{(PH)} \,, \\
&= 1 - \frac{\sqrt{\Omega_\Lambda}}{c} \quad \text{(FH)} \,,
\end{aligned}
\tag{18}
$$

The HH condition implies

$$
\frac{d\Omega_\Lambda}{dx} = 0 \,.
\tag{19}
$$

so that Ω_Λ (and Ω_m) is constant. This scenario has been shown to be inconsistent with the observational data[11] in the absence of any interaction. However as we now proceed to show, if a constant rate of decay Γ of dark energy into matter is assumed, one can recover a good fit to the data.

The PH and FH conditions have been considered more recently, and have been exploited[14, 20] to attempt to achieve a fixed point solution for Ω_Λ. It has been shown that the FH condition is needed if one is to obtain a value of Ω_Λ that is suitably close to the valued obtained from fits to the supernova data. However assuming that Ω_Λ has acquired a value suitably close to its fixed point value does not guarantee a good fit to the data. In fact, by assuming PH or FH as the holographic condition and an interaction as a function on Ω_Λ, the quantity $\dot{H}/H^2$ would be a function of Ω_Λ alone as in Eq. (14). If also Ω_Λ is approximately constant because it is near its fixed point, then the universe is characterized by two components with constnat equations of state w_m^{eff} and w_Λ^{eff}.

Consider the case where a holographic condition associates the dark energy length scale with the Hubble parameter (HH) and the interaction is set equal a constant of the same scale as the Hubble constant,

$$
\frac{\dot{L}_\Lambda}{L_\Lambda} = -\frac{\dot{H}}{H} \,, \qquad \Gamma = \kappa H_0 \,.
\tag{20}
$$

Here we require κ to be an order one constant. As pointed out in Ref. [11] the first condition in Eq. (20) in the absence of any interaction ($\Gamma = 0$) gives rise to a description of the universe inconsistent with the observational data. In this circumstance, the holographic condition implies that the cosmological evolution will appear as matter-dominated ($w = 0$) at all times. However when the interaction (decay) of the dark energy in Eqn. (20) is included, one can obtain a suitable fit. The Hubble parameter satisfies

$$
\frac{H(z)}{H_0} = \left(1 - \frac{\kappa}{3r_0}\right)(1+z)^{3/2} + \frac{\kappa}{3r_0} \,,
\tag{21}
$$

where $r_0 = \Omega_{m,0}/\Omega_{\Lambda,0}$. This evolution of the Hubble parameter exhibits the needed features to agree with observations: for large z a characteristic $(1 + z)^{3/2}$ behavior of a matter-dominated era with an approach at smaller z to a constant $H(z)$ characteristic of a de Sitter phase. A good fit is obtained when $\kappa/(3r_0) \approx 0.62$ (see Section IV). Comparing to the ΛCDM solution in Eq. (16), the best fits result for parameters *roughly* equal to $\Omega_{\Lambda,0} = 2/3$ and $\Omega_{m,0} = 1/3$ in the ΛCDM and similarly $\kappa/(3r_0) = 2/3$ in the interacting dark energy model (the parameter $\kappa/(3r_0)$ in the interacting dark energy model plays an analogous role to that of $\Omega_{\Lambda,0}$ in the ΛCDM). Nevertheless the best fit value for $\kappa/(3r_0)$ in the interacting dark energy model is not equal to the best fit value of $\Omega_{\Lambda,0}$ in the ΛCDM.

The characteristics of the model can be understood from

$$
\frac{\dot{H}}{H^2} = \frac{3}{2}\left(-1 + \frac{\Gamma}{3H}\frac{\Omega_\Lambda}{1-\Omega_\Lambda}\right) + \frac{1}{2(1-\Omega_\Lambda)}\frac{d\Omega_\Lambda}{dx} \,.
\tag{22}
$$

A constant Ω_Λ causes the last term to vanish and the $1/r = \Omega_\Lambda/(1 - \Omega_\Lambda)$ factor is constant and equal to its present value $1/r_0$. Then

$$
\frac{\dot{H}}{H^2} = \frac{3}{2}\left(-1 + \frac{\Gamma}{3H}\frac{1}{r_0}\right) \,,
\tag{23}
$$

Then $H(z)$ clearly exhibits a transition between a matter-dominated era ($\dot{H}/H^2 = -3/2$) and a constant H era ($\dot{H}/H^2 = 0$ when $H_\infty = \kappa H_0/(3r_0)$ for $\Gamma = \kappa H_0$). At early times the interaction is negligible, and the holographic condition enforces an expansion of the Universe that appears as matter-dominated. When the Hubble parameter becomes sufficiently small, the interaction becomes effective and a fixed point solution is reached.

While the interacting dark energy model predicts a constant Ω_Λ the coincidence problem emerges as the choice of scale $\Gamma = \kappa H_0$. The quantity κ is an order one to give a good fit to all observational data.

The holographic condition $\dot{L}_\Lambda/L_\Lambda = -\dot{H}/H$ implies the effective equations of state are equal,

$$
\begin{aligned}
w_m^{\text{eff}} = w_\Lambda^{\text{eff}} &= -\frac{\Gamma}{3H}\frac{\Omega_\Lambda}{(1-\Omega_\Lambda)} \,, \\
&= -\frac{\Gamma}{3Hr_0} \,.
\end{aligned}
\tag{24}
$$

With the interaction $\Gamma = \kappa H_0$ one recognizes that the common equation of state for the matter and dark energy components varies from 0 to -1 as the Hubble parameter decreases from large values in the early Universe to its asymptotic value ($H_\infty = \kappa H_0/(3r_0)$).

IV. QUANTITATIVE COMPARISON OF THE MODEL WITH OBSERVATIONAL DATA

In this section we compare the interacting dark energy model predictions from Eq. (21) with supernova data. Comparison of the observational data with holographic models with *noninteracting* dark energy was performed in Refs. [15] and [19]. The holographic condition based on the Hubble horizon (HH) is grossly inconsistent with the data as shown by Hsu[11], but other holographic models based on the future event horizon (FH) can be made consistent with the data.

The luminosity distance is defined as

$$d_L(z) = (1+z)H_0^{-1}\int_0^z \frac{dz'}{E(z')} \, , \qquad (25)$$

where $E(z) = H(z)/H_0$. In the interacting dark energy model, $E(z)$ is obtained from the integration of Eq. (6). The observational data for the supernovae is expressed in terms of an apparent magnitude and redshift. Assuming the supernovae have the same absolute magnitude M, then the extinction-corrected distance moduli is given by

$$\mu(z) = 5\log_{10}(d_L(z)/\text{Mpc}) + 25 \, . \qquad (26)$$

Using the calculated distance moduli in Eq. (26) for an interacting dark energy model and the supernovae data[43], one can perform a χ-squared fit,

$$\chi^2 = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{mod}}(z_i)]^2}{\sigma_i^2} \, , \qquad (27)$$

where the sum runs over the supernova data points and where the σ_i are the experimental errors in each observation. The ΛCDM model gives a χ^2 of 178 for the 157 data points in the Gold data sample[43] for $\Omega_{m,0} = 0.27$ and $\Omega_{\Lambda,0} = 0.73$.

A comparison of the supernova data with the interacting dark energy model is shown in Fig. 1. Taking the Hubble constant as $H_0^{-1} = 2997.9h^{-1}$ Mpc, the best fit values for r_0/κ is 0.54 and for the Hubble constant is h =0.64. The 1σ and 2σ contours obtained when varying the parameters h and r_0/κ are shown in Fig. 2.

The luminosity distance has dimensions of length and therefore scales with the inverse Hubble constant H_0. This means the model curve for $\mu(z)$ can be shifted up or down by adjusting H_0. The interacting dark energy model is also consistent with CMB measurement of the shift parameter[44, 45, 46, 47] and measurements of large scale structure by SDSS[48]. Since the large z behavior
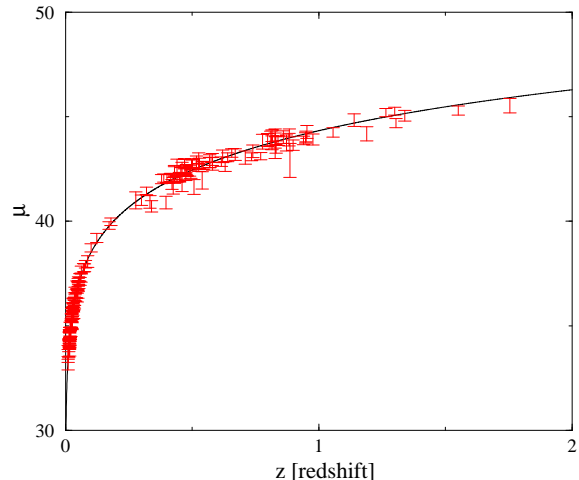

FIG. 1: Comparison of the supernova data with the model prediction for $\mu(z)$ with $r_0/\kappa = 1/2$. The data shown in the Gold data from Ref.[43]. The best fit occurs for h=0.64 for which the χ^2 is 179 for 157 degrees of freedom.

of the interacting dark energy model is the same as the ΛCDM, the predicted value of shift parameter

$$\mathcal{R} = \sqrt{1 - \Omega_\Lambda^0}\int_0^{z_{\text{dec}}} \frac{dz'}{E(z')} \, . \qquad (28)$$

is easily accommmodated with a choice of $r_0/\kappa \approx 1/2$ and $\Omega_{\Lambda,0} \approx 0.7$. The utility of the shift parameter $\mathcal{R}$ and the A-parameter measured by SDSS is that they are independent of H_0[49]. Consequently the gross geometric features of this model agree with the expansion history predicted by the ΛCDM. An important question remains as to whether the detailed observations of the large scale structure can be made consistent with the matter density predicted at early times in this model (or some variant).

V. SUMMARY AND CONCLUSIONS

The ΛCDM model provides an explanation for all observational data. However there remain a number of important issues it must confront. Given our lack of understanding of the dark energy, one can ask if there are other simple physical properties that dark energy might have that could equally well account for the data.

In this paper we have shown that a model of dark energy with a holographic condition relating the dark energy to the Hubble parameter and a constant interaction of size roughly equal to the Hubble constant H_0 can give a good fit to data. This model is characterized by a constant ratio of Ω_m to Ω_Λ. Since the ratio of matter to dark energy is constant due to the holographic condition
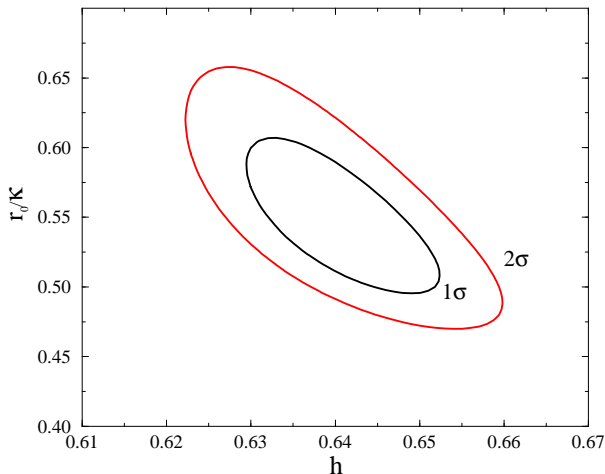
FIG. 2: The $\Delta\chi^2 = 2.30, 6.17$ contours $(1\sigma, 2\sigma)$ for the parameters h and r_0/κ in the interacting dark energy model.

and the effective equations of state are equal, one can

view the evolution of the model as one comprised of one component whose effective equation of state varies between 0 when the decay of the dark energy into matter is negligible to an asymptotic value of -1 when the interaction become important. The transition between one regime and the other occurs when the interaction rate is comparable to the Hubble parameter.

We have shown that the expansion history of the Universe can be reproduced with a model of interacting dark matter. For the specific model presented here the functional dependence $H(z)$ differs slightly between the ΛCDM and the model discussed in this paper. Future data from SNAP may be useful in discriminating between them. Finally information that goes beyond the recorded expansion history of the Universe may be useful for ruling out this kind of model.

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